correspond to including the fact that the moment of inertia depends on the internal coordinate *r*.

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Loop Entropy of the Triple Helix

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ABSTRACT: We calculate the loop entropy in an infinitely long triple-stranded chain and estimate its contribution to the helix–coil transition of collagen by comparing theory with experimental data. We find that the temperature dependence of helix fraction together with that of specific heat provides a rather stringent test for a simple model. By comparing our model (with loops) and the one-sequence model (without loops) with experiment, we find that the formation of loops is responsible for a substantial fraction of heat absorption of collagen during its helix–coil transition.

The calculation of loop entropy in triple-stranded helices is itself an interesting statistics problem. It also has practical applications to helix–coil transitions of collagen (and its analogues). In this paper we will try to calculate the loop entropy in an infinitely long triple-stranded chain and estimate its contribution to the helix–coil transition of collagen by comparing theory with experimental data.

The statistics of loop formation in double-stranded helices has been much studied,[1-4] but the triple-stranded problem is much more complicated. In order to obtain a manageable formula for our final result, we ignore the effect of excluded volume,[3] assume a Gaussian distribution for the end-to-end extension of a loop,[1] and use a seminumerical method to carry out a double summation. Experimental data include the heat absorption and optical activity of tropocollagen measured by Privalov and Tiktopulo.[5] We will find that the temperature dependence of helix fraction together with that of specific heat provides a rather stringent test for a simple model. By comparing our model with the one-sequence model, we can see that the formation of loops is responsible for a substantial fraction of heat absorption of collagen during its helix–coil transition.

Theory

Consider, as a statistical mechanics problem, a homogeneous triple-stranded chain. Each strand is a sequence of basic units. A triple-helix unit consists of three basic units, one from each component strand. When a triple-helix unit becomes unbound, it is called a coil state. At a given temperature, the configuration of a triple-stranded chain is an alternate sequence of helical and loop states. A loop sequence consists of three unbound component strands of various length (number of basic units) attached to two helical sequences at the beginning and at the end.[6] In the limit of infinitely long chain, dissociation is neglected. It is then convenient to use the method of sequence-generating functions[7] to calculate the partition function. We write the sequence-generating function for helical states as[7]

$$V(x) = \sum_{l=1}^{\infty}(S/x^3)^l = S/(x^3 - S) \qquad (1)$$

where S is the partition function for each triple-helix unit and can be written in the form

$$S = \exp\{\Delta H_u(T - T_c)/RTT_c\} \qquad (2)$$

Here ΔH_u and $\Delta H_u/T_c$ are, respectively, the changes of enthalpy and entropy from a *free* coil state (see below) to a helical state. x is a parameter which will be used later to calculate the partition function. The sequence-generating function for loop states is

$$U(x) = \beta\sum_{l=1}^{\infty} u_l/x^l \qquad (3)$$

where u_l is the partition function of a loop with a total strand length l (the total number of basic units in the three strands) and β is the border factor between a loop state and a helical state. If one writes u_l as

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$$u_l = e^{\Delta S_l/R} \tag{4}$$

then ΔS_l is the change of entropy from a free coil state (in which the three component strands are independent) to a loop state (in which the three component strands are attached to two helical sequences at the beginning and at the end). The free coil state is introduced here merely as a convenient reference for calculation.

One approximate way of calculating the loop entropy is to consider a loop as three free strands constrained to a fixed end-to-end distance r. Define $W(r,l)$ as the distribution function for a strand of length l with the end-to-end distance r. $W(r,l)$ is normalized as

$$\int_0^{r_{\max}} W(r,l)\,\mathrm{d}r = 1 \tag{5}$$

If we ignore the effect of excluded volume,[3] it can be shown that[1]

$$\Delta S_l = R \ln \left\{ \sum_{l_1+l_2+l_3=l} \delta(l_2)\delta(l_3)\Delta^2 \int_0^{r_{\max}} W(r,l_1)W(r,l_2)W(r,l_3)\,\mathrm{d}r \right\} \tag{6}$$

In eq 6, δ, defined as the fraction of the solid angle 4π, is the restriction in solid angle for the end-to-end vectors of the second and third strands once the end points of the first strand are fixed. Δ is a similar restriction in radial distance for the second and the third strands. $\delta(l)$ is a function of the strand length l, but Δ is a constant.

We shall now estimate ΔS_l by using the Gaussian distribution[1]

$$W(r,l)\,\mathrm{d}r = \frac{4}{\pi^{1/2}}\frac{r^2}{r_0^{\,3}}e^{-(r/r_0)^2}\,\mathrm{d}r \tag{7}$$

with $r_0 = r_u(2l/3)^{1/2}$, r_u being the length of a basic unit. Since there are two ways of calculating the entropy of a double-stranded loop, i.e., regarding it as a loop of two strands or a closed loop of one strand, one can calculate $\delta(l)$ in terms of l and r_u. The result is[1]

$$\delta(l) \propto l^{-1} \tag{8}$$

Results of Calculations

Even with the assumptions made above, i.e., Gaussian distribution, eq 7, and neglect of the effect of excluded volume, the calculation of ΔS_l, eq 6, is still formidable. Therefore, we shall take a seminumerical approach. As in the case of double-stranded polymers,[2] we assume that u_l varies as

$$u_l = e^{\Delta S_l/R} = Al^{-\gamma} \tag{9}$$

where A and γ are hopefully slowly varying functions of l for large l. As an example, consider the perfect-matching loops in which l_1, l_2, and l_3 of eq 6 are the same and equal to $l/3$. Using eq 7, one can easily show that as $l \to \infty$

$$[\delta(l/3)]^2\Delta^2 \int_0^{r_{\max}} \{W(r,l/3)\}^3\,\mathrm{d}r = (\mathrm{const})l^{-3} \tag{10}$$

This is, of course, only one of the possible terms in eq 6. The latter has the l dependence

$$u_l \propto \sum_{l_1=1}^{l-2}\sum_{l_2=1}^{l-l_1-1} \frac{l_1^{\,2}(l-l_1-l_2)l_2}{\{(l-l_1-l_2)l_2 + l_1(l-l_1)\}^{7/2}} \equiv f(l) \tag{11}$$

in the long-chain limit. Since ΔS_l is the entropy of a loop state minus the entropy of its corresponding free coil state, it is always negative. That means $u_l \leq 1$ or $\gamma > 0$. We rewrite eq 9 and 11 as

$$f(l) \equiv A'l^{-\gamma} \tag{12}$$

Table I
Numerical Calculation of A' and γ (Eq 11 and 12)

l_1	l_2	γ	A'
10	15	0.470	0.0559
100	150	0.907	0.2249
500	510	0.965	0.3154
1000	1050	0.977	0.3401

It can be easily shown that $\gamma \to 1$ as $l \to \infty$.[8] However, for large but finite l, γ is a slowly varying function of l, with a value smaller than 1. This is seen by assuming A' and γ as constants and solving for them by substituting two different values l_1 and l_2 into eq 12. Table I shows the values of A' and γ solved from different sets of l_1 and l_2 in different regions of l. Above $l = 500$, γ and A' vary slowly with l. When we compare the model with the data of collagen ($N \sim 300$), we shall treat γ as a constant between 0.85 and 0.95.[9] A' will be absorbed by another unknown factor β. It turns out that the fitting is not sensitive to the value of γ in that range. The sequence-generating function for loop states can then be written as

$$U(x) = B\sum_{l=1}^{\infty}\frac{1}{x^l}l^{-\gamma} \approx B\frac{\Gamma(2-\gamma)}{\gamma-1}\left[1-\left(\frac{x-1}{x}\right)^{\gamma-1}\right] \tag{13}$$

where B is a constant proportional to β. The approximate equality in eq 13 is valid for $\gamma < 1$.[10]

Comparison with Collagen Data

The partition function for a triple-stranded polymer of length N as $N \to \infty$ is[7]

$$Q = x_1^{\,3N} \tag{14}$$

if x_1 is the root of the secular equation

$$U(x)V(x) = 1 \tag{15}$$

or

$$\frac{\Gamma(2\gamma)}{\gamma-1}\left[1-\left(\frac{x-1}{x}\right)^{\gamma-1}\right] = \frac{x^3-S}{BS} \tag{16}$$

The fraction of helical states θ is given by

$$\theta = \frac{1}{N}\frac{\partial \ln Q}{\partial \ln S} = 3\frac{S}{x_1}\frac{\partial x_1}{\partial S} \tag{17}$$

and the specific heat by

$$C = 3NRT\frac{\partial^2}{\partial T^2}\{T \ln x_1\} \tag{18}$$

In the transition region, we have $T \sim T_c$. Therefore S is close to 1 (see eq 2). In eq 16 we note that (1) $x_1 > 1$, (2) $1 - \gamma \sim 0.1$, $[(x-1)/x]^{\gamma-1} \sim 1$, and, hence, (3) $x_1 = 1 + O(B)$. B is proportional to the border factor. In the range of the transition region, its temperature dependence is weak and will be ignored. The border factor is very small (10^{-2} or less) in the single-stranded and double-stranded polymers.[10] Thus we look for the root x_1 in a region close to 1. Another simplification can be made for the specific heat, eq 18, in the transition region. For $T \sim T_c$, we have

$$S \approx \exp\left[\frac{\Delta H_u}{RT_c^{\,2}}(T-T_c)\right] \tag{19}$$

and consequently

$$C \approx \frac{N\Delta H_u T}{T_c^{\,2}}\left(2\theta + T\frac{\partial\theta}{\partial T}\right) \tag{20}$$

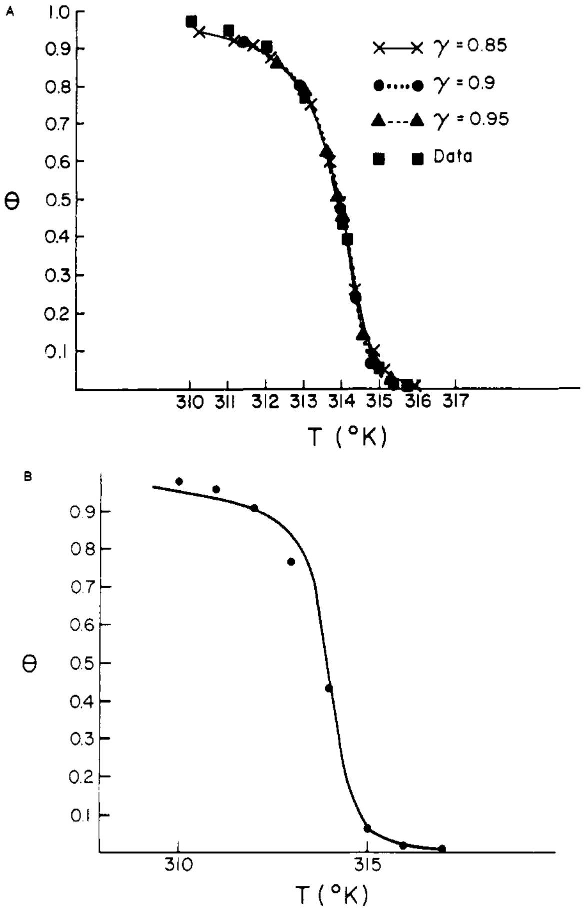

Figure 1. (A) Fraction of helical states, θ, as a function of temperature. The data are from ref 5. The theoretical curves of our model are shown at three different γ's. (B) Comparison of the one-sequence model (solid curve) with data (dots). The data are the same as those in Figure 1A.

The best fits with Privalov and Tiktopulo's data at γ = 0.85, 0.90, and 0.95 are shown in Figures 1A and 2A with the following parameters:

γ	T_c[a]	ΔH_u[b]	B
0.85	313.2	−14.1	1.1×10^{-2}
0.90	313.1	−13.2	1.7×10^{-2}
0.95	313.0	−13.8	1.4×10^{-2}

[a] In K. [b] In kcal/(mol of triple-helix unit).

Conclusions

To help evaluate these fittings, we consider another long-chain model which also has three adjustable parameters, i.e., the one-sequence model. The partition function for this model is

$$Q = 1 + b\sum_{l=1}^{N} S^l (N - l + 1)^3 \qquad (21)$$

where S has the same meaning as in eq 1 and b is the appropriate border factor. The fraction of helical states is then given as

$$\theta = \frac{1}{N}\frac{\partial \ln Q}{\partial \ln S} \qquad (22)$$

and the specific heat as

$$C = RT\frac{\partial^2}{\partial T^2}(T \ln Q) \qquad (23)$$

The three adjustable parameters are ΔH_u and T_c in S (see

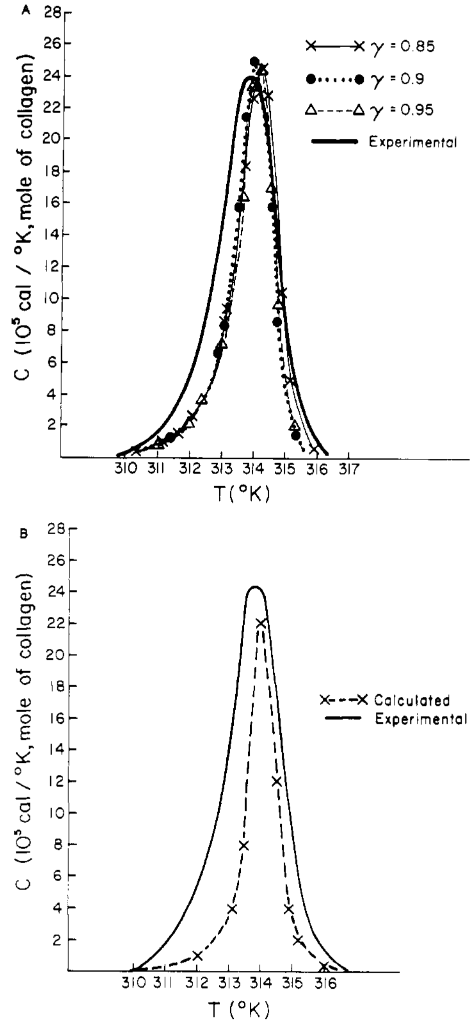

Figure 2. (A) Specific heat, C, as a function of temperature. The data are from ref 5. The theoretical curves of our model are shown at three different γ's. (B) Comparison of the one-sequence model (the calculated curve) with data (solid curve). The data are the same as those in Figure 2A.

eq 2) and b. The best fit to Privalov and Tiktopulo's data is shown in Figures 1B and 2B with the following values for ΔH_u, T_c, and b:

$$\Delta H_u = -9.35 \text{ kcal/(mol of triple-helix unit)}$$

$$T_c = 314.5 \text{ K} \qquad (24)$$

$$b = 1.93 \times 10^{-9}$$

We see that although the one-sequence model can fit the θ curve, it can only account for about 45% of the heat absorption (area under the specific heat curve).

In comparison, the model including loops accounts for better than 80% of the heat absorption. The deviation between our model and the data can be attributed to at least two reasons: the inhomogeneity and the finite length of natural collagen. Nevertheless it seems clear that loops are important for the heat absorption of collagen.

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(8) Goldstein, B.; DeLisi, C. *J. Chem. Phys.* **1969**, *51*, 431. *Ibid.* **1970**, *52*, 476. These papers discuss a model similar to ours but assume $\gamma = 1$. With this value of γ, the subsequent formulas for θ and C are greatly simplified. However, we found that these θ and C give a very poor fit to the data.
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(10) Reference 4, p 223.

Studies on the Radical Chain Copolymerization of Methyl Methacrylate and Styrene at Their Azeotropic Composition

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ABSTRACT: In this work the radical chain copolymerization of methyl methacrylate and styrene at their azeotropic composition is studied. The propagation and termination reactions for this copolymerization are described by the apparent rate expressions. The apparent rate constants of propagation and termination, k_{pa} and k_{ta}, determined at low extent of reaction are 334.0 L mol^{-1} s^{-1} and 10.45×10^7 L mol^{-1} s^{-1}, respectively, and the Φ value obtained is 15. The latter coincides with that obtained when a chemically-controlled termination model is assumed. The relative importance of combination in the termination is evaluated as 68.2%. When the monomer conversion reaches 11 mol %, the monomer conversion vs. reaction time curve predicted with the k_{pa} and k_{ta} values mentioned above starts to deviate from the experimental data. The deviation is attributed to a gel effect. Thus, the Hamielec equation is used in correlating k_{ta}. We find that $k_{ta} = [10.45 \times 10^7/(1 - X)^2] \exp(0.412X - 12.50X^2)$ and $k_{pa} = 334.0$ L mol^{-1} s^{-1} can satisfactorily describe the reaction course of the conversion vs. reaction time for this copolymerization.

Introduction

In free radical polymerization the termination reaction has been proposed to occur by following three steps:[1] (1) the two macromolecules diffuse together so that it becomes possible for the two radical chain ends to move into proximity; (2) segmental rearrangement occurs so as to bring the free-radical sites into a position in which chemical change can occur; (3) chemical change takes place by either a combination or a disproportionation process. If the first two steps are controlling steps, the reaction will be diffusion controlled. If, on the other hand, the third step is the controlling step, the reaction becomes chemically controlled. North et al.[2-4] claim that for most cases the termination reaction is diffusion controlled even in the most mobile solvents available. They present a single value of the rate constant k_{t12} to describe the termination reaction between like radicals and between unlike radicals. Walling,[5] Bevington et al.,[6] and Fukui et al.[7] adopt the chemically-controlled model to study the copolymerization of methyl methacrylate with styrene and introduce a factor Φ ($k_{t12}/2(k_{t1}k_{t2})^{1/2}$) to explain the cross-termination reaction of this copolymerization. Eastmond[8] recalculated the data of Bevington et al.,[6] assuming a single value of k_{t12}, and found that, even when termination is diffusion controlled, a single value of k_{t12} cannot provide an adequate description of the termination process and that some preference for the cross-termination reaction still exists. Russo and Munari[9] adopt the approach that it is only the flexibility of the chain end which is important physically, rather than the overall chain composition and flexibility. They propose the existence of termination reactions to take account of both chemical and physical effects of the penultimate unit. It is then assumed that the rate constants for the cross termination are given by the geometric means of appropriation of homotermination reactions. Finally, they are able to describe the copolymerization of styrene with methyl methacrylate.

In this work the copolymerization of methyl methacrylate and styrene (MMA–St) at their azeotropic composition is studied. It is known that the types of rate expressions and the values of rate constants depend on the mechanism chosen for the reaction. However, as described above, the kinetic treatment on the termination reaction is still somewhat controversial. In this study we do not start by giving a kinetic scheme of the reaction as most standard texts do but represent all rate equations of radical chain copolymerization by the apparent rate expressions, regardless of the true reaction mechanism. The equations for determining the rate constants of propagation and termination and the relative importance of combination and disproportionation in the overall termination reaction are established. Then, following the possible reaction mechanisms, we work out relationships between the apparent rate constants and all rate constants involved in the elementary reactions. Finally, we compare the theoretical and observed values of the apparent rate constants and discuss the mechanism of the termination reaction for this azeotropic copolymerization.

Theory

1. Aspects of Apparent Rate Coefficients of Propagation and Termination. With the application of the long-chain hypothesis, we introduce the apparent rate expression for the disappearance of the two types of monomers, A and B, for the free-radical copolymerization

$$R_p = -dM/dt = k_{pa}X_TM \qquad (1)$$

where k_{pa} denotes the apparent rate constant of propagation. M refers to the total concentrations of monomers A and B in the reaction mixture at reaction time t, and

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